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Thermodynamics function of a strongly degenerate bose gas →

The thermodynamics of a strongly degenerate Bose gas refers to a system of bosons at temperatures at or below the quantum degeneracy temperature, where quantum statistics dominate and Bose - Einstein condensation (BEC) occurs.

Below is a structured explanation of the key thermodynamic function.

1) condition for strong Degeneracy →

A Bose gas becomes strongly degenerate when the thermal de Broglie wavelength is comparable to or larger than the interparticle spacing

$$n \lambda_T^3 \gtrsim 1$$

where

$$\lambda_T = \sqrt{\frac{2\pi\hbar^2}{mK_B T}}$$

The critical temperature for Bose - Einstein condensation is

$$T_c = \frac{2\pi\hbar^2}{mK_B}\left(\frac{n}{\zeta(3/2)}\right)^{2/3}$$

wher

$$\zeta(3/2) \approx 2.612$$

For strong degeneracy

(7) Entropy $\rightarrow$

$$S \propto T^{3/2}$$

More Precisely

$$S = \frac{5}{2} \zeta(5/2) \frac{V}{\lambda_T^3} K_B$$

Entropy vanishes as $T \to 0$, consistent with the third law.

(8) Summary of low-Temperature Behavior ($T << T_c$)

Quantity	Temperature Dependence
chemical Potential	$\mu \simeq 0$
condensate fraction	$1 - (T/T_c)^{3/2}$
Internal energy	$U \propto T^{5/2}$
Pressure	$P \propto T^{5/2}$
Specific heat	$C_V \propto T^{3/2}$
Entropy	$S \propto T^{3/2}$

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More precisely

$$U = \frac{3}{2} \zeta(5/2) \frac{V}{\lambda_T^3} K_B T$$

For $T << T_c$:

$$U \propto T^{5/2}$$

5) Pressure $\rightarrow$

$$P = \frac{2}{3} \frac{U}{V}$$

Thus:

$$P \propto T^{5/2}$$

unlike a Fermi gas, Pressure goes to zero as $T \rightarrow 0$.

6) Specific Heat $\rightarrow$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V$$

since $U \propto T^{5/2}$

$$C_V \propto T^{3/2}$$

so:

$$C_V = \frac{15}{4} N K_B \left(\frac{T}{T_c}\right)^{3/2}$$

As $T \rightarrow 0$

$$C_V \rightarrow 0$$

$T << T_c$

(2) chemical Potential $\Rightarrow$

For an ideal Bose gas

- At $T > T_c$: $\mu < C$
- At $T \leq T_c$:

$$\mu \to 0$$

For $T << T_c$:

$$\mu \approx 0$$

(3) condensate fraction $\rightarrow$

Below T_c:

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_c}\right)^{3/2}$$

For strong degeneracy

$$\frac{N_0}{N} \approx 1$$

Most Particles occupy the ground state.

(4) Internal Energy $\rightarrow$

only excited particles contribute to energy

$$U = \frac{3}{2} N K_B T \left(\frac{T}{T_c}\right)^{3/2}$$